Theory of Activated Transport in Bilayer Quantum Hall Systems

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We analyze the transport properties of bilayer quantum Hall systems at total filling factor $\nu = 1$ in drag geometries as a function of interlayer bias, in the limit where the disorder is sufficiently strong to unbind meron-antimeron pairs, the charged topological defects of the system. We compute the typical energy barrier for these objects to cross incompressible regions within the disordered system using a Hartree-Fock approach, and show how this leads to multiple activation energies when the system is biased. We then demonstrate using a bosonic Chern-Simons theory that in drag geometries, current in a single layer directly leads to forces on only two of the four types of merons, inducing dissipation only in the drive layer. Dissipation in the drag layer results from interactions among the merons, resulting in very different temperature dependences for the drag and drive layers, in qualitative agreement with experiment.

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Double layer quantum Hall systems at filling factor $\nu = 1$ display many properties akin to those of superfluids [1]. This behavior results from the pairing of electrons in one layer with holes in the other, producing excitons that condense into a state with interlayer coherence even in the absence of tunneling [2]. In experiments these systems display a strong interlayer tunneling peak at zero bias, reminiscent of the DC-Josephson effect [3], and vanishing single-layer resistances as temperature $T \to 0$ in counterflow experiments [4]. Nevertheless, the "superfluidity" in this system remains *imperfect*: there is no truly dissipationless transport at low but finite temperature in either of these types of experiments. Recently, it has been demonstrated that this behavior may be qualitatively understood if one assumes that disorder produces unpaired merons – the analog of vortices in a thin film superfluid – that remain weakly mobile at any finite temperature [5].

While these experiments strongly suggest the near-coherence of the two layers in this system, one class of experiments has so far defied explanation in these terms. These are drag measurements, in which current is injected and removed in a single layer, and the voltage drop measured in either layer. The resulting resistances when measured as a function of temperature have roughly activated behaviors. The activation energies for the drive and drag layers behave very differently with interlayer bias: the former are *asymmetric* with respect to the bias direction, while the latter are roughly symmetric [6, 7]. Naïvely this suggests that each layer has separate quasiparticles with different activation energies, and interlayer coherence essentially plays no role. Yet such a picture is very difficult to reconcile with the experiments described above, in which imperfect superfluidity is manifest.

In this paper, we will propose a solution to this puzzle. Our approach involves a transport theory for this system in which disorder is incorporated [5] via a slowly-varying random potential (such as results from a remote doping layer), producing puddles of charged quasiparticles [8]. In the context of quantum Hall bilayers, one expects such quasiparticles to be constructed from topological defects [1]. These *merons* carry a vorticity ($s = \pm 1$) in the relative phase of the wavefunction amplitude for each layer, and an interlayer polarization ($\sigma = \pm 1$) arising from a charge imbalance in the meron core that may tilt into either of the two layers. Moreover, because of the remarkable properties of electrons in a single Landau level [9], the *topological* charge of such objects is tied to their *physical* charge: $q_{s\sigma} = -s\,\sigma\,\nu_{-\sigma}$, with $\nu_{+1} = \nu_U$ and $\nu_{-1} = \nu_L$, and $\nu_{U(L)}$ the filling factor in the upper (lower) layer. This will have important consequences for their response to currents.

In this "coherence network" model [5], most of the area is taken up by puddles where both the interlayer coherence and the incompressibility are lost due to the large local density of merons. However, puddles are separated by narrow strips of incompressible Hall fluid [8] with local filling factor very close to 1. Thus, any measured activation energy actually reflects the energy barrier for a meron to cross such an incompressible strip in moving from one puddle to its neighbor. Below, we describe Hartree-Fock (HF) calculations to estimate the dependence of this energy barrier on interlayer bias. Our results demonstrate that the activation barrier depends significantly on the relative orientation of the polarization of merons and the bias. This explains the multiple activation energies in quantum Hall bilayers.

To understand the transport properties of the merons, we map the system to a bosonic superfluid state using a Chern-Simons (CS) transformation, in which each electron is understood as a *boson* with a single unit of mag-

netic flux directed opposite to the applied magnetic field [9, 10]. At mean-field level the magnetic field is canceled, and the quantum Hall state may be understood as a Bose condensate of the composite bosons. In such a description, the quantum Hall bilayer is condensed in *two* senses: with respect to the (bosonic) charge degrees of freedom, and with respect to the interlayer degree of freedom. In computing the total force on a meron due to currents in the system, one must account for forces due to the excess charge on a meron, with which there is an associated magnetic flux, and due to its vorticity in the interlayer phase. In the case of current in just one layer these forces cancel *precisely* for merons with polarization directed toward the current-carrying layer, so that only half the merons move in direct response to the current. Moreover, the induced voltage due to motion of the driven merons turns out to lie *completely* in the driven layer. Then the induced voltage depends on activation barriers for merons of a *single* polarization, leading to an asymmetric activation energy with respect to bias, as seen in experiment.

A voltage drop is induced in the drag layer only through interactions: a driven meron may pair with an undriven meron of opposite polarization (to form a bimeron [1]) over some distance, inducing a voltage in the drag layer. Since the relevant activation energy now involves the crossing of merons of *both* polarizations over incompressible strips, one expects the resulting activation energy to be symmetric with bias. This again is the experimental observation.

Numerical calculation of energies: In equilibrium, one expects the energy of merons residing in different puddles to be roughly equal. When a meron hops from one puddle to the next it must pass through an incompressible strip, where its energy is higher. Computing the activation energy directly for such a process is challenging because it is difficult to fully model the effects of the puddles. In what follows we will estimate the electrostatic (Hartree) contribution to the activation energy, and demonstrate that it is sensitive to the interlayer bias.

Our approach is a Green's function equation of motion method [11] whose application to meron-antimeron states of a quantum Hall bilayer has been described elsewhere [12, 13]. The method generates order parameters $\rho_{ij}(\mathbf{G}) = \frac{1}{N_\phi}\sum_{X,X'} e^{-\frac{i}{2}G_x(X+X')}\delta_{X,X'-G_y\ell^2}\langle c^\dagger_{X,i}c_{X',j}\rangle$, where $c^\dagger_{X,i}$ creates an electron in layer i in a lowest Landau level (LLL) state with guiding center quantum number X, for Hartree-Fock states with crystalline order, characterized by a set of reciprocal lattice vectors $\{\mathbf{G}\}$. To inject merons and antimerons into this state, one works slightly above or below filling factor 1. (We use $\nu = 1.02$ in the results described here). In the limit of very small tunneling, the lowest energy state of the system is a square lattice, with merons at the center and corner of the unit cells, and antimerons at the face centers. The results can be conveniently expressed in terms of a pseudospin vector $\mathbf{S}$, defined by $S_x + iS_y = \rho_{12}$ and $S_z = \rho_{11} - \rho_{22}$. In this language a meron lattice is a complicated non-collinear ferromagnetic state. Fig. 1 illustrates S_z for a typical such lattice. Note that the polarizations of the merons, represented by the sign of S_z at their cores, is opposite for merons and antimerons, so that the charge of the two objects is the same.

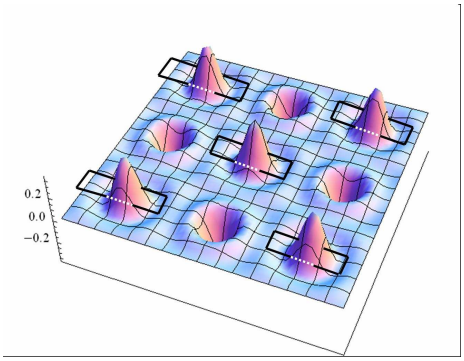

FIG. 1: The z-component of pseudospin density (in units of $1/2\pi\ell^2$) in a square lattice of merons with two electrons per unit cell at $d/\ell = 1.0$ and $\nu = 1.02$. Black lines are schematic depiction of the imposed barrier potentials.

While the puddles of merons largely screen the local disorder potential, inside the incompressible strips there is no such screening. Moreover, because of their high density of merons, the pseudospin stiffness inside the puddles is significantly compromised relative to that of the incompressible strips separating them. Thus the maximum energy configuration for a meron crossing a strip will occur when the meron is centered upon it. (Note that the strip widths are of order ℓ [5, 8], which is narrow compared to the meron size.) Our HF approach allows us to investigate the effect of interlayer bias on the first of these contributions to the energy barrier. To do this, we add an external "box" potential of height V_0 and width ℓ along finite strips in the unit cell, forming a checkerboard pattern (see Fig. 1). These potential strips represent the difference in electrostatic energy for charge located in a strip and charge located in a puddle. By adjusting the position of the meron lattice relative to the external potential, we can arrange for the potential to be directly beneath the $\sigma = +1$ merons, the $\sigma = -1$ merons, or between merons. The energy difference between the "on-meron" and "off-meron" configurations yields an estimate of the electrostatic energy barrier for a meron of a particular polarization σ to cross a strip. In practice, we adjust the barrier height V_0 so that this energy difference matches the activation energy in Ref. 6 for unbiased wells. We can then examine how this activation energy evolves as the layers are biased.

Fig. 2 illustrates some typical results. Merons with polarization oriented in the direction opposite that of

the bias have an *increasing* activation energy with bias, while those with polarization in the same direction *decrease* in energy. (Data for $\Delta\nu < 0$ were generated using $\Delta\nu > 0$ results, which is valid due to a symmetry upon interchange of the layers.) The result may be understood qualitatively by noticing that the charge of the former (latter) increases (decreases) with bias, leading to a larger (smaller) electrostatic energy cost for traversing the barrier. Thus, as observed in experiment and has been so challenging to explain, one obtains two activation energies with opposite slopes as a function of $\Delta\nu$.

The left inset of Fig. 2 illustrates the slope of such curves at $\Delta\nu = 0$ for different layer separations. The apparent non-monotonic behavior may be understood as follows. As d increases from small values, the merons become smaller [1] so that a larger portion of their area lies in the barrier region as they cross. When d becomes still larger, the interlayer exchange interaction decreases enough that the meron charge density deforms when centered on the barrier. With less charge in the barrier region, the sensitivity to bias decreases. This this behavior is a testable prediction of our model.

Finally, the right inset of Fig. 2 illustrates the activation energy at zero bias for fixed d and various values of field B ($\ell \propto \sqrt{1/B}$). One may see that the activation energy in our model decreases with increasing B, as is the general trend in experiment.

The slope of the activation energy as a function of bias in our model is smaller than that found in experiment [6], leading to an activation energy about a factor of 2 smaller than what is observed experimentally at 10% polarization. Although there may be several reasons for such a discrepancy, a large component of it is likely to come from the absence of quantum fluctuations in our model. This compromises the pseudospin stiffness in the incompressible region, and since the experiments are operated near the coherent-incoherent transition for the system [6], the effect should be considerable. Application of bias is known to *increase* coherence effects in bilayers [14], implying that the stiffness in the barrier region should significantly increase with bias. This will increase the sensitivity of the activation energy barrier to the bias.

Meron Dynamics: Having seen how merons may display multiple activation energies when the bilayer is biased, we now turn to a description of their dynamics and the associated dissipation. We work in the composite boson picture, in which the electron system at $\nu = 1$ is modeled as a Bose condensate in zero average magnetic field, with an additional degree of freedom, the pseudospin.

In general, any uniform current in a bilayer system can be decomposed into counterflow (CF) and co-flow currents. Using the superfluid analogy, CF may be described by spatial rotation of the order parameter phase, which in the pseudospin description is the argument of $S_x + iS_y$. This same phase angle may alternatively be interpreted as the condensate phase when the interlayer coherent state is described as an exciton condensate [2]. Because of the vorticity associated with a meron, such currents induce Magnus forces. In addition, since merons are charged objects, in the composite boson picture they carry residual magnetic flux. Coflow current therefore induces a Lorentz force on a meron in direct analogy to flux lines in superconductors [15]. A non-zero net force on a meron will induce motion, which in turn creates a voltage in each layer. Since the effective excess flux carried by a meron is $q\Phi_0$, with $\Phi_0 = hc/e$, the Lorentz force from coflow current may be written [15] as

$$\vec{F} = (\frac{q}{c}\Phi_0)\vec{J} \times \hat{z}, \tag{1}$$

in which $\vec{J}$ is the sum of upper and lower layer current density, $\vec{J}_U + \vec{J}_L$. To compute the CF force, we need to know the effective velocity of the condensed particle-hole pairs, relative to the average velocity of all the electrons [10], $v_s = 2\pi\ell^2(J_U + J_L)$. Without loss of generality, one may assume $\nu_L < \nu_U$ so that the exciton velocity in the lab frame, v_{ex}, is the electron velocity in the lower layer. Then in the frame comoving with the average electron velocity, the CF current density is $J_{CF}^{com} = \frac{\nu_L}{2\pi\ell^2}(v_{ex} - v_s) = J_L - \nu_L(J_L + J_U)$. Since the meron contains a full unit of vorticity of the excitonic condensate phase, the force due to CF current becomes

$$\vec{F}_{CF} = hs\vec{J}_{CF}^{com} \times \hat{z} \equiv (es\Phi_0/c)\vec{J}_{CF}^{com} \times \hat{z} \tag{2}$$

in which s is the vorticity. Adding (1) and (2) and using $q = -s\,\sigma\,\nu_{-\sigma}$ yields the total force on a meron [16],

$$\vec{F}_T = \frac{es}{2}\Phi_0[(1+\sigma)\vec{J}_L - (1-\sigma)\vec{J}_U] \times \hat{z}. \tag{3}$$

From this expression, it is immediately apparent that merons of a given polarization $\sigma = \pm 1$ respond *only* to the current in a single layer.

Connection to Experiment: The force $F_{s,\sigma}$ on merons of vorticity s and polarization σ will cause them to flow with a velocity $u_{s,\sigma} = \mu_{s,\sigma}F_{s,\sigma}$ where $\mu_{s,\sigma}$ is an effective mobility, which we expect to be thermally activated, with a bias dependence of the activation energy as discussed above. We use the resulting motion of the vortices to find the voltage drops between two points a distance y_0 apart along the direction of electron current. From the Josephson relation, due to the vorticity in the interlayer phase an interlayer voltage drop ΔV is induced when merons pass between these two points [5],

$$\Delta V = \Delta V_U - \Delta V_L = -\frac{2\pi h}{e} y_0 \sum_{s,\sigma} n_{s\sigma} s u_{s\sigma}, \tag{4}$$

where $n_{s,\sigma}$ is the meron density. On the other hand, the CS magnetic flux moving with the merons between the two points induces a potential drop between electrons near the two points that is independent of the layer in

which they reside. This leads to the condition

$$(\nu_U \Delta V_U + \nu_L \Delta V_L) = -\frac{h}{e} y_0 \sum_{s,\sigma} n_{s\sigma} q_{s\sigma} u_{s\sigma}. \qquad (5)$$

In a drag geometry we have, for example, $J_L = 0$ and $\vec{J}_U = \frac{I}{W}\hat{y}$, with I the total current and W the sample width. Combining Eqs. 4 and 5, we obtain $\Delta V_L = 0$ and

$$\frac{\Delta V_U}{I} = \frac{y_0}{W} h\Phi_0 (n_{1,-1}\mu_{1,-1} + n_{-1,-1}\mu_{-1,-1}). \qquad (6)$$

Notice the final result depends on the mobility of *only* merons with polarization $\sigma = -1$. It immediately follows that the voltage drop in the drive layer is asymmetric with respect to bias, precisely as observed in experiment.

In order to explain the voltage drop in the drag layer ($\Delta V_L \neq 0$) we must identify how forces on the $\sigma = +1$ merons might arise. A natural candidate for this is the attractive interaction between merons with opposite vorticities, which in the absence of disorder binds them into pairs at low meron densities. Assuming that driven merons crossing incompressible strips will occasionally be a component of these bimerons, a voltage drop in the drag layer will result. The mobility of such bimerons is limited by the energy barrier to cross an incompressible strip. Since these strips are narrow compared to the size scale of the constituents of the bimeron, we expect the activation energy to be given approximately by the maximum of the activation energies for merons of the two polarizations $\sigma = \pm 1$. This leads to a drag resistance much smaller than that of the drive layer, and an activation energy that is *symmetric* with respect to bias. Both these behaviors are observed in experiment [18].

It is interesting that, within our model, as samples become increasingly clean one expects such mobile bimerons to become more prominent relative to single merons, so that the voltage drop in the two layers will increasingly match with decreasing disorder and decreasing temperature. In principle a drag experiment in a sample clean and cold enough that all merons are paired will result in precisely the same voltage drop in each layer, so that pure counterflow becomes dissipationless, and coflow dissipation occurs in a manner such that one cannot distinguish whether the electrons are moving in the upper or lower layer. Similar behavior should ensue with increased interlayer tunneling, for which merons and antimerons become more strongly bound into pairs, forming the quasiparticles of the system (bimerons) [1, 12, 13].

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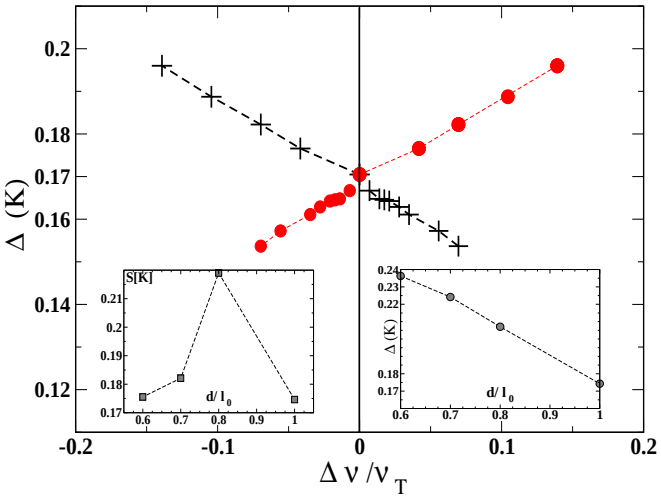

FIG. 2: Activation barrier for merons as a function of layer imbalance for layer separation $d = 0.8\ell$ and $B = 1T$. Barrier height is $0.0061e^2/\ell$. Circles indicate merons with positive polarization, + symbols are for merons with negative polarization. Left inset: Slope $S = d\Delta/d(\Delta\nu)$ of activation energy vs. d at zero bias. Right inset: Activation energy vs. d/ℓ at zero bias, for fixed d ($d = \ell$ at $B = 1T$).

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